

Prof. Dr. Alfred Toth

Reduzierte Situationsklassen

1. Wir gehen wiederum aus von der allgemeinen Form semiotischer Dualsysteme

$$DS: ZKI = (3.x, 2.y, 1.z) \times RTh = (z.1, y.2, x.3),$$

bilden sie auf ihre situationalen Trajektklassen, kurz Situationsklassen (vgl. Toth 2025a) genannt, ab

$$\begin{array}{l} 3_A.x_A \quad \underline{2}_R.y_R \quad 1_I.z_I \quad \rightarrow \quad 3_A.\underline{2}_R \quad x_A.y_R \quad | \quad \underline{2}_R.1_I \quad y_R.z_I \\ z_A.1_A \quad y_R.\underline{2}_R \quad x_I.3_I \quad \rightarrow \quad z_A.y_R \quad 1_A.\underline{2}_R \quad | \quad y_R.x_I \quad \underline{2}_R.3_I \end{array}$$

und erhalten damit folgendes Trajekt-Dualsystem:

$$DST: ZKI^T = (3_A.\underline{2}_R, x_A.y_R | \underline{2}_R.1_I, y_R.z_I) \times RTh^T = (z_A.y_R, 1_A.\underline{2}_R | y_R.x_I, \underline{2}_R.3_I)$$

mit $Sit = (x_A.y_R | \underline{2}_R.1_I)$, $U^{lo} = (3_A.\underline{2}_R)$ und $U^{ro} = (y_R.z_I)$.

Reduzierte Situationsklassen entstehen aus Situationsklassen, indem zusätzlich die Umgebungen U^{lo} und U^{ro} verschränkt werden (vgl. Toth 2025b).

2. Semiotische Klassen, Situationsklassen und reduzierte Situationsklassen im vollständigen ternären semiotischen 27er-System

1. Dualsystem

$$3.1 \quad 2.1 \quad 1.1 \quad \times \quad 1.1 \quad 1.2 \quad 1.3 \quad (1.1 \leftarrow (1.2, 1.3))$$

$$3_A.\underline{2}_R \quad 1_A.\underline{1}_R \quad | \quad \underline{2}_R.1_I \quad \underline{1}_R.1_I$$

$$1_A.\underline{1}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{1}_R.1_I \quad \underline{2}_R.3_I$$

$$((1_A.1_R | 2_R.1_I), (3_A.1_R | 2_R.1_I)) \times ((1_A.2_R | 1_R.1_I), (1_A.2_R | 1_R.3_I))$$

2. Dualsystem

$$3.1 \quad 2.1 \quad 1.2 \quad \times \quad 2.1 \quad 1.2 \quad 1.3 \quad (2.1 \leftarrow (1.2, 1.3))$$

$$3_A.\underline{2}_R \quad 1_A.\underline{1}_R \quad | \quad \underline{2}_R.1_I \quad \underline{1}_R.2_I$$

$$2_A.\underline{1}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{1}_R.1_I \quad \underline{2}_R.3_I$$

$$((1_A.1_R | 2_R.1_I), (3_A.1_R | 2_R.2_I)) \times ((1_A.2_R | 1_R.1_I), (2_A.2_R | 1_R.3_I))$$

3. Dualsystem

$$3.1 \quad 2.1 \quad 1.3 \quad \times \quad 3.1 \quad 1.2 \quad 1.3 \quad (3.1 \leftarrow (1.2, 1.3))$$

$$3_A.\underline{2}_R \quad 1_A.\underline{1}_R \quad | \quad \underline{2}_R.1_I \quad \underline{1}_R.3_I$$

$$3_{A.1R} \quad 1_{A.2R} \quad | \quad \underline{1}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.1R} | 2_{R.1I}), (3_{A.1R} | 2_{R.3I})) \times ((1_{A.2R} | 1_{R.1I}), (3_{A.2R} | 1_{R.3I}))$$

4. Dualsystem

$$3.1 \quad 2.2 \quad 1.1 \quad \times \quad 1.1 \quad 2.2 \quad 1.3 \quad (1.1 \rightarrow 2.2 \leftarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.1I}$$

$$1_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.2R} | 2_{R.1I}), (3_{A.2R} | 2_{R.1I})) \times ((1_{A.2R} | 2_{R.1I}), (1_{A.2R} | 2_{R.3I}))$$

5. Dualsystem

$$3.1 \quad 2.2 \quad 1.2 \quad \times \quad 2.1 \quad 2.2 \quad 1.3 \quad ((2.1, 2.2) \rightarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.2I}$$

$$2_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.2R} | 2_{R.1I}), (3_{A.2R} | 2_{R.2I})) \times ((1_{A.2R} | 2_{R.1I}), (2_{A.2R} | 2_{R.3I}))$$

6. Dualsystem

$$3.1 \quad 2.2 \quad 1.3 \quad \times \quad 3.1 \quad 2.2 \quad 1.3 \quad (3.1 \rightarrow 2.2 \leftarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.3I}$$

$$3_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.2R} | 2_{R.1I}), (3_{A.2R} | 2_{R.3I})) \times ((1_{A.2R} | 2_{R.1I}), (3_{A.2R} | 2_{R.3I}))$$

7. Dualsystem

$$3.1 \quad 2.3 \quad 1.1 \quad \times \quad 1.1 \quad 3.2 \quad 1.3 \quad (1.1 \rightarrow 3.2 \leftarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.3R} \quad | \quad \underline{2}_{R.1I} \quad \underline{3}_{R.1I}$$

$$1_{A.3R} \quad 1_{A.2R} \quad | \quad \underline{3}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.3R} | 2_{R.1I}), (3_{A.3R} | 2_{R.1I})) \times ((1_{A.2R} | 3_{R.1I}), (1_{A.2R} | 3_{R.3I}))$$

8. Dualsystem

$$3.1 \quad 2.3 \quad 1.2 \quad \times \quad 2.1 \quad 3.2 \quad 1.3 \quad (2.1 \rightarrow 3.2 \leftarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.3R} \quad | \quad \underline{2}_{R.1I} \quad \underline{3}_{R.2I}$$

$$2_{A.3R} \quad 1_{A.2R} \quad | \quad \underline{3}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.3R} | 2_{R.1I}), (3_{A.3R} | 2_{R.2I})) \times ((1_{A.2R} | 3_{R.1I}), (2_{A.2R} | 3_{R.3I}))$$

9. Dualsystem

$$3.1 \quad 2.3 \quad 1.3 \quad \times \quad 3.1 \quad 3.2 \quad 1.3 \quad ((3.1, 3.2) \rightarrow 1.3)$$

$$3_{A.2R} \quad 1_{A.3R} \quad | \quad \underline{2}_{R.1I} \quad \underline{3}_{R.3I}$$

$$3_{A.3R} \quad 1_{A.2R} \quad | \quad \underline{3}_{R.1I} \quad \underline{2}_{R.3I}$$

$$((1_{A.3R} | 2_{R.1I}), (3_{A.3R} | 2_{R.3I})) \times ((1_{A.2R} | 3_{R.1I}), (3_{A.2R} | 3_{R.3I}))$$

10. Dualsystem

$$3.2 \quad 2.1 \quad 1.1 \quad \times \quad 1.1 \quad 1.2 \quad 2.3 \quad ((1.1, 1.2) \rightarrow 2.3)$$

$$3_{A.2R} \quad 2_{A.1R} \quad | \quad \underline{2}_{R.1I} \quad \underline{1}_{R.1I}$$

$$1_{A.1R} \quad 1_{A.2R} \quad | \quad \underline{1}_{R.2I} \quad \underline{2}_{R.3I}$$

$$((2_{A.1R} | 2_{R.1I}), (3_{A.1R} | 2_{R.1I})) \times ((1_{A.2R} | 1_{R.2I}), (1_{A.2R} | 1_{R.3I}))$$

11. Dualsystem

$$3.2 \quad 2.1 \quad 1.2 \quad \times \quad 2.1 \quad 1.2 \quad 2.3 \quad (2.1 \rightarrow 1.2 \leftarrow 2.3)$$

$$3_{A.2R} \quad 2_{A.1R} \quad | \quad \underline{2}_{R.1I} \quad \underline{1}_{R.2I}$$

$$2_{A.1R} \quad 1_{A.2R} \quad | \quad \underline{1}_{R.2I} \quad \underline{2}_{R.3I}$$

$$((2_{A.1R} | 2_{R.1I}), (3_{A.1R} | 2_{R.2I})) \times ((1_{A.2R} | 1_{R.2I}), (2_{A.2R} | 1_{R.3I}))$$

12. Dualsystem

$$3.2 \quad 2.1 \quad 1.3 \quad \times \quad 3.1 \quad 1.2 \quad 2.3 \quad (3.1 \rightarrow 1.2 \leftarrow 2.3)$$

$$3_{A.2R} \quad 2_{A.1R} \quad | \quad \underline{2}_{R.1I} \quad \underline{1}_{R.3I}$$

$$3_{A.1R} \quad 1_{A.2R} \quad | \quad \underline{1}_{R.2I} \quad \underline{2}_{R.3I}$$

$$((2_{A.1R} | 2_{R.1I}), (3_{A.1R} | 2_{R.3I})) \times ((1_{A.2R} | 1_{R.2I}), (3_{A.2R} | 1_{R.3I}))$$

13. Dualsystem

$$3.2 \quad 2.2 \quad 1.1 \quad \times \quad 1.1 \quad 2.2 \quad 2.3 \quad (1.1 \leftarrow (2.2, 2.3))$$

$$3_{A.2R} \quad 2_{A.2R} \quad | \quad \underline{2}_{R.1I} \quad \underline{2}_{R.1I}$$

$$1_{A.2R} \quad 1_{A.2R} \quad | \quad \underline{2}_{R.2I} \quad \underline{2}_{R.3I}$$

$$((2_{A.2R} | 2_{R.1I}), (3_{A.2R} | 2_{R.1I})) \times ((1_{A.2R} | 2_{R.2I}), (1_{A.2R} | 2_{R.3I}))$$

14. Dualsystem

$$3.2 \quad 2.2 \quad 1.2 \quad \times \quad 2.1 \quad 2.2 \quad 2.3 \quad (2.1 \leftarrow (2.2, 2.3))$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 2_A \cdot \underline{2}_R & | & \underline{2}_R \cdot 1_I & \underline{2}_R \cdot 2_I \\
2_A \cdot \underline{2}_R & 1_A \cdot \underline{2}_R & | & \underline{2}_R \cdot 2_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((2_A \cdot 2_R | 2_R \cdot 1_I), (3_A \cdot 2_R | 2_R \cdot 2_I)) \times ((1_A \cdot 2_R | 2_R \cdot 2_I), (2_A \cdot 2_R | 2_R \cdot 3_I))$$

15. Dualsystem

$$\begin{array}{ccc}
3.2 & 2.2 & 1.3 \quad \times \quad 3.1 \quad 2.2 \quad 2.3 \quad (3.1 \leftarrow (2.2, 2.3))
\end{array}$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 2_A \cdot \underline{2}_R & | & \underline{2}_R \cdot 1_I & \underline{2}_R \cdot 3_I \\
3_A \cdot \underline{2}_R & 1_A \cdot \underline{2}_R & | & \underline{2}_R \cdot 2_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((2_A \cdot 2_R | 2_R \cdot 1_I), (3_A \cdot 2_R | 2_R \cdot 3_I)) \times ((1_A \cdot 2_R | 2_R \cdot 2_I), (3_A \cdot 2_R | 2_R \cdot 3_I))$$

16. Dualsystem

$$\begin{array}{ccc}
3.2 & 2.3 & 1.1 \quad \times \quad 1.1 \quad 3.2 \quad 2.3 \quad (1.1 \rightarrow 3.2 \leftarrow 2.3)
\end{array}$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 2_A \cdot \underline{3}_R & | & \underline{2}_R \cdot 1_I & \underline{3}_R \cdot 1_I \\
1_A \cdot \underline{3}_R & 1_A \cdot \underline{2}_R & | & \underline{3}_R \cdot 2_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((2_A \cdot 3_R | 2_R \cdot 1_I), (3_A \cdot 3_R | 2_R \cdot 1_I)) \times ((1_A \cdot 2_R | 3_R \cdot 2_I), (1_A \cdot 2_R | 3_R \cdot 3_I))$$

17. Dualsystem

$$\begin{array}{ccc}
3.2 & 2.3 & 1.2 \quad \times \quad 2.1 \quad 3.2 \quad 2.3 \quad (2.1 \rightarrow 3.2 \leftarrow 2.3)
\end{array}$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 2_A \cdot \underline{3}_R & | & \underline{2}_R \cdot 1_I & \underline{3}_R \cdot 2_I \\
2_A \cdot \underline{3}_R & 1_A \cdot \underline{2}_R & | & \underline{3}_R \cdot 2_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((2_A \cdot 3_R | 2_R \cdot 1_I), (3_A \cdot 3_R | 2_R \cdot 2_I)) \times ((1_A \cdot 2_R | 3_R \cdot 2_I), (2_A \cdot 2_R | 3_R \cdot 3_I))$$

18. Dualsystem

$$\begin{array}{ccc}
3.2 & 2.3 & 1.3 \quad \times \quad 3.1 \quad 3.2 \quad 2.3 \quad ((3.1, 3.2) \rightarrow 2.3)
\end{array}$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 2_A \cdot \underline{3}_R & | & \underline{2}_R \cdot 1_I & \underline{3}_R \cdot 3_I \\
3_A \cdot \underline{3}_R & 1_A \cdot \underline{2}_R & | & \underline{3}_R \cdot 2_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((2_A \cdot 3_R | 2_R \cdot 1_I), (3_A \cdot 3_R | 2_R \cdot 2_I)) \times ((1_A \cdot 2_R | 3_R \cdot 2_I), (2_A \cdot 2_R | 3_R \cdot 3_I))$$

19. Dualsystem

$$\begin{array}{ccc}
3.3 & 2.1 & 1.1 \quad \times \quad 1.1 \quad 1.2 \quad 3.3 \quad ((1.1, 1.2) \rightarrow 3.3)
\end{array}$$

$$\begin{array}{cc|cc}
3_A \cdot \underline{2}_R & 3_A \cdot \underline{1}_R & | & \underline{2}_R \cdot 1_I & \underline{1}_R \cdot 1_I \\
1_A \cdot \underline{1}_R & 1_A \cdot \underline{2}_R & | & \underline{1}_R \cdot 3_I & \underline{2}_R \cdot 3_I
\end{array}$$

$$((3_A.1_R | 2_R.1_I), (3_A.1_R | 2_R.1_I)) \times ((1_A.2_R | 1_R.3_I), (1_A.2_R | 1_R.3_I))$$

20. Dualsystem

$$3.3 \quad 2.1 \quad 1.2 \quad \times \quad 2.1 \quad 1.2 \quad 3.3 \quad (2.1 \rightarrow 1.2 \leftarrow 3.3)$$

$$3_A.\underline{2}_R \quad 3_A.\underline{1}_R \quad | \quad \underline{2}_R.1_I \quad \underline{1}_R.2_I$$

$$2_A.\underline{1}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{1}_R.3_I \quad \underline{2}_R.3_I$$

$$((3_A.1_R | 2_R.1_I), (3_A.1_R | 2_R.2_I)) \times ((1_A.2_R | 1_R.3_I), (2_A.2_R | 1_R.3_I))$$

21. Dualsystem

$$3.3 \quad 2.1 \quad 1.3 \quad \times \quad 3.1 \quad 1.2 \quad 3.3 \quad (3.1 \rightarrow 1.2 \leftarrow 3.3)$$

$$3_A.\underline{2}_R \quad 3_A.\underline{1}_R \quad | \quad \underline{2}_R.1_I \quad \underline{1}_R.3_I$$

$$3_A.\underline{1}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{1}_R.3_I \quad \underline{2}_R.3_I$$

$$((3_A.1_R | 2_R.1_I), (3_A.1_R | 2_R.3_I)) \times ((1_A.2_R | 1_R.3_I), (3_A.2_R | 1_R.3_I))$$

22. Dualsystem

$$3.3 \quad 2.2 \quad 1.1 \quad \times \quad 1.1 \quad 2.2 \quad 3.3 \quad (1.1 \rightarrow 2.2 \leftarrow 3.3)$$

$$3_A.\underline{2}_R \quad 3_A.\underline{2}_R \quad | \quad \underline{2}_R.1_I \quad \underline{2}_R.1_I$$

$$1_A.\underline{2}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{2}_R.3_I \quad \underline{2}_R.3_I$$

$$((3_A.2_R | 2_R.1_I), (3_A.2_R | 2_R.1_I)) \times ((1_A.2_R | 2_R.3_I), (1_A.2_R | 2_R.3_I))$$

23. Dualsystem

$$3.3 \quad 2.2 \quad 1.2 \quad \times \quad 2.1 \quad 2.2 \quad 3.3 \quad ((2.1, 2.2) \rightarrow 3.3)$$

$$3_A.\underline{2}_R \quad 3_A.\underline{2}_R \quad | \quad \underline{2}_R.1_I \quad \underline{2}_R.2_I$$

$$2_A.\underline{2}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{2}_R.3_I \quad \underline{2}_R.3_I$$

$$((3_A.2_R | 2_R.1_I), (3_A.2_R | 2_R.2_I)) \times ((1_A.2_R | 2_R.3_I), (2_A.2_R | 2_R.3_I))$$

24. Dualsystem

$$3.3 \quad 2.2 \quad 1.3 \quad \times \quad 3.1 \quad 2.2 \quad 3.3 \quad (3.1 \rightarrow 2.2 \leftarrow 3.3)$$

$$3_A.\underline{2}_R \quad 3_A.\underline{2}_R \quad | \quad \underline{2}_R.1_I \quad \underline{2}_R.3_I$$

$$3_A.\underline{2}_R \quad 1_A.\underline{2}_R \quad | \quad \underline{2}_R.3_I \quad \underline{2}_R.3_I$$

$$((3_A.2_R | 2_R.1_I), (3_A.2_R | 2_R.3_I)) \times ((1_A.2_R | 2_R.3_I), (3_A.2_R | 2_R.3_I))$$

25. Dualsystem

3.3 2.3 1.1 × 1.1 3.2 3.3 (1.1 ← (3.2, 3.3))

$3_{A.2R}$ $3_{A.3R}$ | $2_{R.1I}$ $3_{R.1I}$

$1_{A.3R}$ $1_{A.2R}$ | $3_{R.3I}$ $2_{R.3I}$

$((x_A, y_R | 2_{R.1I}), (3_{A.y_R} | 2_{R.z_I})) \times ((1_{A.2R} | y_{R.x_I}), (z_A.2_R | y_{R.3_I}))$

26. Dualsystem

3.3 2.3 1.2 × 2.1 3.2 3.3 (2.1 ← (3.2, 3.3))

$3_{A.2R}$ $3_{A.3R}$ | $2_{R.1I}$ $3_{R.2I}$

$2_{A.3R}$ $1_{A.2R}$ | $3_{R.3I}$ $2_{R.3I}$

$((3_{A.3R} | 2_{R.1I}), (3_{A.3R} | 2_{R.2I})) \times ((1_{A.2R} | 3_{R.3I}), (2_{A.2R} | 3_{R.3I}))$

27. Dualsystem

3.3 2.3 1.3 × 3.1 3.2 3.3 (3.1 ← (3.2, 3.3))

$3_{A.2R}$ $3_{A.3R}$ | $2_{R.1I}$ $3_{R.3I}$

$3_{A.3R}$ $1_{A.2R}$ | $3_{R.3I}$ $2_{R.3I}$

$((3_{A.3R} | 2_{R.1I}), (3_{A.3R} | 2_{R.3I})) \times ((1_{A.2R} | 3_{R.3I}), (3_{A.2R} | 3_{R.3I}))$

Literatur

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